A nonblocking approach for the parallel computation of the Laplacian

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- It exploits the peculiar features of the Laplacian operator to get a more efficient parallel implementation

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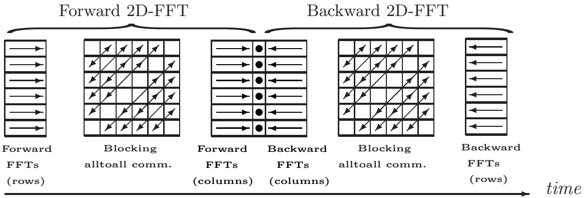

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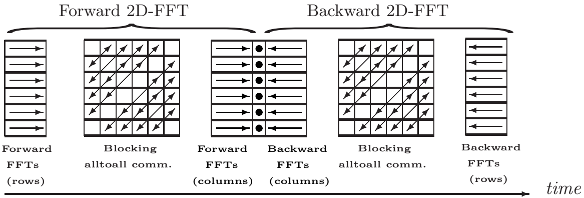

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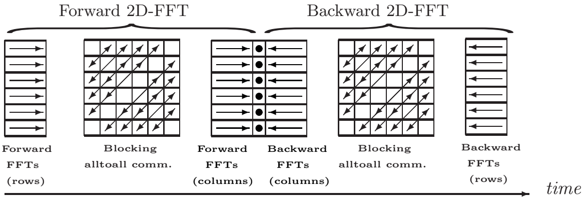

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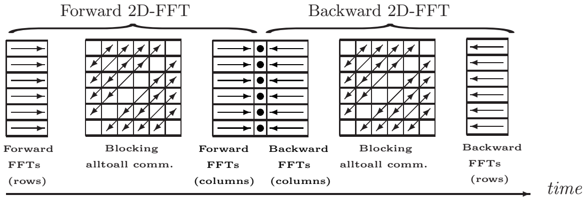

- Four computation steps separated by communications
- Dimensions are sequentially computed in a divide-and-conquer strategy
- Blocking communications assure such strict sequencing

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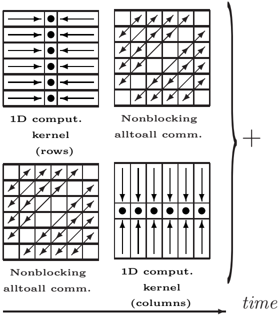

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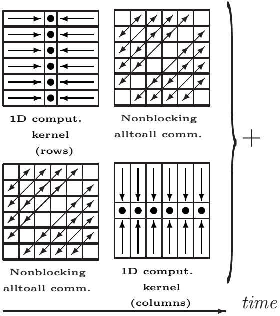

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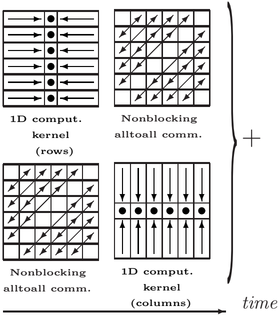

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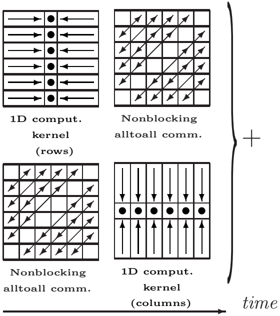

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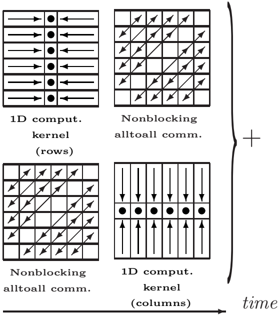

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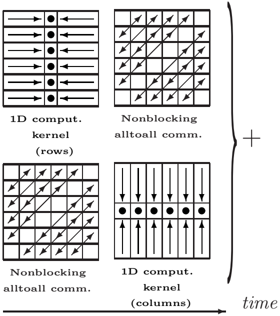

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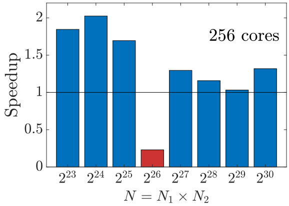

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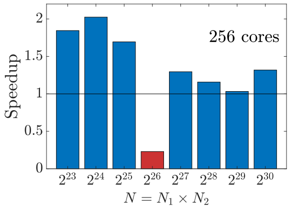

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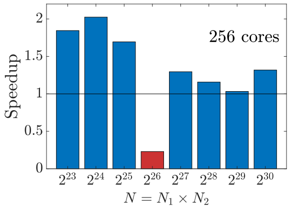

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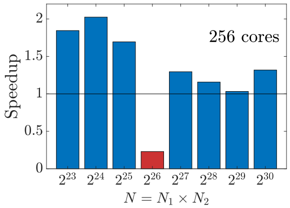

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- the `FFTW-MPI` replicates the cache miss for $N/P^2 = 2^{11}$

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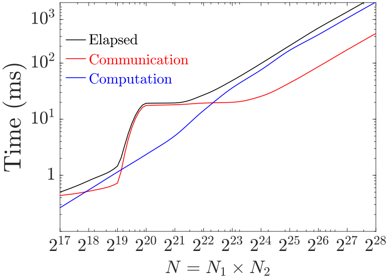

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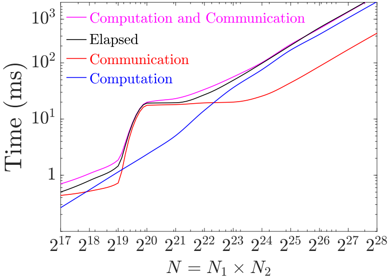

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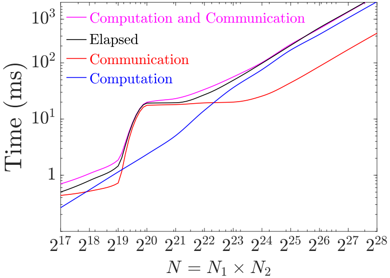

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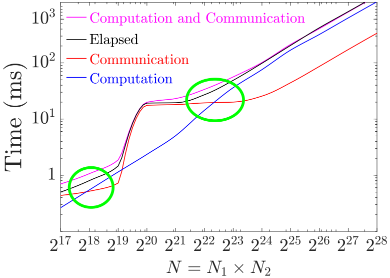

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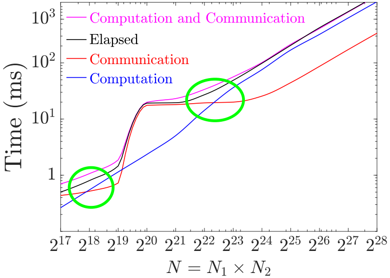

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- However, no benefits are achieved when one prevails over the other

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7. Main programmer task: find this scenario (if possible) and work around it